数理解析研究所講究録
971 巻 1996 年 116-124

On Demuškin Groups

Maurice Arrigoni*
Tokyo Metropolitan University

Abstract

Let k be an algebraic number field, p an odd prime, and $G_S = \mathrm{Gal}(k_S/k)$, where k_S is the maximal pro-p-extension of k unramified outside the set S of primes dividing p. We consider situations where G_S is a Demuškin group, especially when k is totally real or of CM type.

Several authors gave conditions in order to express the Galois group G_S as a free pro-p-product:

$$G_S = (\mathop{\star}_{v\in T} G_v) \star F$$

where F is a free pro-p-group, T is a subset of S and $G_v = \mathrm{Gal}(k_v(p)/k_v)$, $k_v(p)$ being the maximal pro-p-extension of the localization of k at v. However these conditions are rather restrictive, implying in particular that the decomposition group G_S^v of an extension of $v \in T$ to k_S is equal to the whole local group G_v. Keeping this in mind, we are all the more interested in the cases where G_S is a Demuškin group without being equal to G_v for any $v \in S$.

When k is totally real, we can give a criterion for G_S to be a Demuškin group involving Iwasawa theory, see Proposition 5. Using numerical data, we obtain many examples when $p = 3$ and k is a real quadratic field.

We classify the Demuškin groups G_S in two types, depending on the existence of a place $v \in S$ such that the natural map

$$H^2(G_S, \mathbb{Z}/p\mathbb{Z}) \to H^2(G_v, \mathbb{Z}/p\mathbb{Z})$$

is an isomorphism. If such a v exists, we say that G_S is a *Demuškin group of local type.*

When k is totally real or of CM type, we give necessary and sufficient conditions for G_S to be a Demuškin group of local type, involving only the arithmetic of the base field k, see Theorems 1 and 2. Examples are given in the case $p = 3$ and k is an imaginary bi-quadratic field. In these examples we have $G_S = G_S^v$, where v is the unique place of k dividing 3 and $G_S \neq G_v$; this was the situation we were interested in, without allowing the case k is totally real where the Proposition 5 gives an easy characterization.

---

*This research was partly supported by Grant-in-Aid for Scientific Research, Ministry of Education, Science, Sports and Culture, Japanese Government, No. 06094224.

1 Introduction

Let us introduce some notations associated with the field k and the prime p. We denote by
— r_1 (resp. r_2) the number of real (resp. complex) archimedean places of k
— k_S the maximal pro-p-extension of k unramified outside S
— k_∞ the cyclotomic $\mathbb{Z}_p$-extension of k ($k_\infty \subset k_S$), $\Gamma = \mathrm{Gal}(k_\infty/k)$ and $\Lambda = \mathbb{Z}_p[[\Gamma]]$
— μ_p the group of p-th roots of unity, and for a field F, $\mu(F)$ the p-primary part of the group of roots of unity contained in F
— k_v the localization of k at a prime v, U_v (resp. $U_v^{(1)}$) the group of units (resp. principal units) in the ring of integers of k_v and $n_v = (k_v : \mathbb{Q}_\ell)$ the local degree, where v divides the prime ℓ
— $k_v(p)$ the maximal pro-p-extension of k_v
— $G_S = \mathrm{Gal}(k_S/k)$, $G_v = \mathrm{Gal}(k_v(p)/k_v)$
— G_S^v the decomposition group of an extension of v to k_S (G_S^v is defined up to an inner automorphism of G_S)
— $k' = k(\mu_p)$, $\Delta = \mathrm{Gal}(k'/k)$
— $\delta = \begin{cases} 1 & \text{if } \mu_p \subset k \\ 0 & \text{otherwise} \end{cases}$ $\quad \delta_v = \begin{cases} 1 & \text{if } \mu_p \subset k_v \\ 0 & \text{otherwise} \end{cases}$
— E the unit group of the ring of integers of k
— Cl (resp. Cl_S) the p-Sylow of the class group (resp. S-class group) of k
— $V_S = \{a \in k^\times \mid a \in k_v^{\times p} \text{ for } v \in S;\ a \in U_v k_v^{\times p} \text{ for } v \notin S\}/k^{\times p}$. By comparing class field theory and Kummer theory, we obtain an isomorphism

$$V_S \simeq \mathrm{Hom}_\Delta(\mathrm{Cl}_S(k'), \mu_p) \tag{1}$$

When there is more than one algebraic number field in consideration, we will append the name of the field to the above objects.
Let G be a pro-p-group. For every integer n, let $H^n(G) = H^n(G, \mathbb{Z}/p\mathbb{Z})$, G acting trivially on $\mathbb{Z}/p\mathbb{Z}$. $d(G) := \dim H^1(G)$ is the generator rank of G, and $r(G) := \dim H^2(G)$ is the relation rank of G.

A motivation for our problem is given by the following local result

1. If $\mu_p \not\subset k_v$, then G_v is a free pro-p-group.
2. If $\mu_p \subset k_v$, then G_v is a Demuškin group.

Definition 1 *G is defined to be a Demuškin group if $d(G)$ is finite, $r(G) = 1$, and the cup-product*

$$H^1(G) \times H^1(G) \to H^2(G) \textit{ is a perfect pairing.}$$

Let us now consider the global situation. The Galois group G_S has the following well-known property:

cd(G_S) (the cohomological dimension of G_S) is ≤ 2, and $d(G_S)$ is finite.

and this property holds also for a Demuškin group. After the case G_S is a free pro-p-group (or equivalently $cd(G_S) = 1$), the case G_S is a Demuškin group might be considered as the easiest situation to handle in restricted ramification Theory.

In order to evaluate the relation rank of G_S, the Poitou-Tate duality gives the following exact sequence

$$0 \to \mu_p(k) \to \prod_{v \in S} \mu_p(k_v) \to H^2(G_S)^* \to V_S \to 0 \tag{2}$$

where A^* denotes the Pontrjagin dual of a locally compact abelian group. In the sequence (2), the local group $\mu_p(k_v)$ is in duality with $H^2(G_v)$ and the map $\mu_p(k_v) \to H^2(G_S)^*$ corresponds to the composition $G_v \to G_S^v \to G_S$.

We will use the following relation between the generator rank and the relation rank of G_S giving the value of its Euler-Poincaré characteristic:

$$\chi(G_S) := 1 - d(G_S) + r(G_S) = -r_2 \tag{3}$$

Let us suppose that G_S is a Demuškin group. Then the relation rank of G_S is equal to one, so there are only two possibilities:

1. $V_S = 0$ and $\delta + 1 = \sum_{v \in S} \delta_v$ —we will say that G_S is a *Demuškin group of local type*. Then the natural map $H^2(G_S) \to H^2(G_v)$ is an isomorphism, for every $v \in S$ such that $\delta_v = 1$.

2. $\dim V_S = 1$ and $\delta = \sum_{v \in S} \delta_v$ —we will say that G_S is a *Demuškin group of global type*. Then, $H^2(G_S) \to H^2(G_v)$ is the zero map for every $v \in S$.

In any case there are at most two primes $v \in S$ such that $\delta_v = 1$. Furthermore such primes can not split in k_S; otherwise there would exist a finite extension K in k such that $\sum_{v \in S(K)} \geq p > 2$, which is absurd because $G_S(K)$, having finite index in G_S, is also a Demuškin group. So we have proved the following

Proposition 1 *Suppose that G_S is a Demuškin group and there exists $v \in S$ such that $\delta_v = 1$. Then $G_S = G_S^v$.*

2 Known Results

We quote here some results relevant to our problem.

Suppose first that k contains μ_p and G_S is a Demuškin group. Then, by Proposition 1, the places of S do not split in k_S and we have $|S| = 1$ (resp. $|S| = 2$) if G_S is a Demuškin group of global (resp. local) type. Conversely, if k contains μ_p, using a result of Kuz'min, see [4, Fondamental Theorem, A & B(a)], we have

1. if $|S(k_S)| = 1$, then G_S is a free pro-p-group.

2. if $|S(k_S)| = 2$, then $G_S = G_{v_1} = G_{v_2}$, where $S(k) = \{v_1, v_2\}$, hence G_S is a Demuškin group.

Hence, we deduce the following (essentially due to Kuz'min)

Proposition 2 *Suppose that k contains μ_p and $S = S_p$. Then G_S is a Demuškin group if and only if $S = \{v_1, v_2\}$ and v_i does not split in k_S for $i = 1, 2$. Moreover, we have $G_S = G_{v_1} = G_{v_2}$, and G_S is a Demuškin group of local type.*

Ku'zmin gave the following example: Let $p = 3$ and $k = \mathbb{Q}(\sqrt{-3}, \sqrt{15})$. Then G_S is a Demuškin group.

In [6], Wingberg gives necessary and sufficient conditions for G_S to be equal, up to a free pro-p-group F, to the free pro-p-product of the local groups G_v for $v \in (S \setminus S_0)$, where $S_0 \subset S$ is a maximal subset with the property (+):

$$G_S = (\mathop{\star}_{v \in (S \setminus S_0)} G_v) \star F \qquad \text{with } (+): \sum_{w \in S_0} \delta_w = \delta$$

In the particular case of Demuškin groups , here are the conditions (see loc. cit. Theorem (i) and Corollary b):

Proposition 3 *Let $v \in S$ such that $\delta_v = 1$. Then $G_S = G_v$ (hence G_S is a Demuškin group of local type) if and only if*

$$S_0 = S \setminus \{v\} \textit{ has the property (+)}, \; r_2 = n_v, \textit{ and } V_{S_0}^S = 0,$$

where $V_{S_0}^S = \{a \in k^\times \mid a \in k_v^{\times p} \text{ for } v \in S_0;\ a \in U_v k_v^{\times p} \text{ for } v \notin S\}/k^{\times p}$.

Remark. Except the case $\delta = 1$ studied by Kuz'min, there is no example of field k satisfying Proposition 3.

3 Totally real fields

In order to tackle the totally real case, we will use the following result, see [1, Prop. 1]

Proposition 4 *Let G be a pro-p-group having a closed normal subgroup H such that the quotient $\Gamma = G/H$ is a one-generator free pro-p-group. Then the following conditions are equivalent*

1. *G is a two-generator Demuškin group.*

2. *H is a one-generator free pro-p-group.*

We suppose now that the field k is totally real. Then by the equality (3), we have $d(G_S) = 1 + r(G_S)$. Hence, by the previous Proposition, G_S is a Demuškin group if and only if the group $H = \mathrm{Gal}(k_S/k_\infty)$ is isomorphic to $\mathbb{Z}_p$, and from a property of pro-p-groups, this is true if and only if the maximal abelian pro-p-quotient H^{ab} is isomorphic to $\mathbb{Z}_p$. We take the following notations of Iwasawa's theory:

- $H = \mathrm{Gal}(k_S/k_\infty)$, $\mathcal{X} = H^{ab}$ the maximal Λ-module unramified outside S
- k_n the n-th layer of the cyclotomic extension k_∞/k
- $X = \varprojlim \mathrm{Cl}(k_n)$, $X' = \varprojlim \mathrm{Cl}_S(k_n)$, the projective limits being taken via the norm maps. By class field theory, X (resp. X') is isomorphic to the Galois group of the maximal abelian pro-p-extension of k_∞ which is unramified (resp. unramified and where the places of $S(k_\infty)$ split totally).
- $\Delta \xrightarrow{\omega} \mathbb{Z}_p^\times$ the Teichmüller describing the action of Δ on $\zeta \in \mu_p$: $\sigma(\zeta) = \zeta^{\omega(\sigma)}$. For a $\mathbb{Z}_p[\Delta]$-module M, we denote by M_ω the ω-component of M:

$$M_\omega = \{m \in M \mid \delta.m = \omega(a)m \text{ for every } \delta \in \Delta\}$$

- $\lambda(M)$ and $\mu(M)$ the Iwasawa invariants of a Noetherian Λ-module M.

We use the following standard results of Iwasawa's theory, see for instance [3]

> $\mathcal{X}(k)$ and $X(k')_\omega$ are Noetherian torsion Λ-modules having the same λ and μ invariants, and $\mathcal{X}$ has no finite submodule other than 0.

This implies that $\mu(\mathcal{X}) = 0$ if and only if $\mathcal{X}$ is a free $\mathbb{Z}_p$-module, and in that case the $\mathbb{Z}_p$-rank of $\mathcal{X}$ is equal to $\lambda(\mathcal{X})$. Hence, we obtain:

Proposition 5 *Let k be a totally real field . Then G_S is a Demuškin group if and only if $\mu(X(k')_\omega) = 0$ and $\lambda(X(k')_\omega) = 1$.*

Example. Let $k = \mathbb{Q}(\mu_p)^+$ be the maximal real subfield of $\mathbb{Q}(\mu_p)$. Then $\mu(X(k'))$ is equal to 0 by the Theorem of Ferrero-Washington, and for $p < 125000$, $\lambda(X(k')_\omega$ is equal to the irregularity index $i(p)$ of p, see [5, Remark following Corollary 10.3]. So for $p < 125000$, G_S is a Demuškin group if and only if $i(p) = 1$. In this case, the unique place $v \in S$ is totally ramified in k_S, because $\mathrm{Cl}(k) = 0$ (*Vandiver's Conjecture*) is true for $p < 125000$. In particular we have $G_S = G_S^v$, although G_S is a Demuškin group of global type.

We will now consider the case where G_S is a Demuškin group of local type for a totally real field k. The discussion at the end of the introduction shows that we must suppose that there exists a unique $v \in S$ such that $\delta_v = 1$ and v does not split in any S-ramified extension of k. We have the following criterion using only the arithmetic of k, see [1, Th. 1]:

Theorem 1 *Let k be a totally real field, having a unique $v \in S$ such that $\delta_v = 1$, and suppose that v does not split in k_∞/k. Then the following conditions are equivalent:*

1. *G_S is a Demuškin group (of local type).*
2. *Let $w(F)$ be the cardinal of $\mu(F)$ for a field F, and let h, R_p, D be respectively the class number, the p-adic regulator, and the discriminant of k. Then*

$$\frac{w(k')hR_p}{w(k_v)\sqrt{D}} \prod_{w\in S} (1 - Norm_{k_w/\mathbb{Q}_p} w^{-1}) \text{ is a p-adic unit.}$$

3. $X'(k')_\omega = 0$

4. The map $E/E^p \to \prod_{w\in S} U_w/U_w^p$ *is injective, and the p-Hilbert class field of k (the maximal abelian unramified pro-p-extension of k) is contained in* k_∞.

Example. $k = \mathbb{Q}(\sqrt{d})$ is a real quadratic field, and $p = 3$
We suppose that $d > 1$ is square free integer. Let $k^* = \mathbb{Q}(\sqrt{-m})$ the "mirror field", where $m = d/3$ if 3 divides d, and $m = 3d$ otherwise. In standard way, we have $X(k')_\omega = X(k^*)$, and also similar results for $X'(k^*)$, $\mathrm{Cl}(k^*)$, $\mathrm{Cl}_S(k^*)\ldots$

By the Theorem of Ferrero-Washington, we have $\mu(X(k^*)) = 0$. Hence by Proposition 5, we obtain:

$G_S(k)$ is a Demuškin group $\iff \lambda(X(k^*)) = 1$

We use the tables of [2] giving the λ-invariant of $X(k^*)$ for $0 < m < 100000$. The case $\lambda = 1$ happens quite often (23489 times), giving us a good provision of Demuškin groups. We give examples in two cases, just to illustrate different situations which were discussed: $d \equiv -3 \pmod 9$, and $d \equiv 2 \pmod 3$.

$\underline{d \equiv -3 \pmod 9}$ In this case, 3 is ramified in k, and $\mu_3 \subset k_v$, where $\{v\} = S$. Here are the first ten values of $m \equiv -1 \pmod 3$ for which $\lambda(X(k^*)) = 1$: $m = 2$, 5, 11, 17, 23, 26, 29, 38, 53, 59. Hence

$G_S(\mathbb{Q}\sqrt{d})$ is a Demuškin group of local type for $d = 6$, 15, 33, 51, 69, 78, 87, 114, 159, 177...

It is also possible to use the last condition of Theorem 1: Here, k_∞/k is totally ramified at v, hence the 3-Hilbert class field of k is disjoint to k_∞. So we obtain:

Proposition 6 *Let* $k = \mathbb{Q}\sqrt{d}$*, where d is a positive square free integer such that* $d \equiv -3 \pmod 9$*, and let* $p = 3$*. Then* $G_S(k)$ *is a Demuškin group (of local type) if and only if* $Cl(k) = 0$ *and the fundamental unit* ϵ *of k is not a cube in* U_v*.*

$\underline{d \equiv 2 \pmod 3}$ In this case, 3 remains prime in k, and for every integer n, the unique ideal of k_n dividing 3 is principal, hence $\mathrm{Cl}(k_n) = \mathrm{Cl}_S(k_n)$, which implies $X(k) = X'(k)$. Here are the first values of d such that G_S is a Demuškin group:

$G_S(\mathbb{Q}\sqrt{d})$ is a Demuškin group of global type for $d = 29$, 74, 113, 122, 131, 137, 173, 182, 206, 251, 254, 257...

4 CM fields

In this section, we generalize a process used by Kuz'min to construct Demuškin groups, see [4, Proposition 5.4], where he considered an extension k/k^+ of CM type, the field k containing μ_p. In order to find new examples , we make the following weaker hypothesis:

k/k^+ is an extension of CM type, with Galois group $J = \mathrm{Gal}(k/k^+)$.

For a $\mathbb{Z}_p[J]$-module M, we denote by M^+ and M^- respectively the invariant and anti-invariant part of M under the action of J.

Taking the inflation followed by the restriction, for every integer i, we have isomorphisms

$$H^i(G_S(k^+)) \xrightarrow{\text{inf}} H^i(\mathrm{Gal}(k_S/k^+)) \xrightarrow{\text{res}} H^i(G_S(k))^+$$

for the following reasons: the order of J is prime to p, so the restriction maps are isomorphisms. The isomorphisms hold trivially for $i = 0$, and also for $i \geq 3$ because the cohomology groups vanish. The inflation $H^1(G_S(k^+)) \to H^1(\mathrm{Gal}(k_S/k^+))$ is an isomorphism, because $G_S(k^+)$ is the maximal pro-p-quotient of $\mathrm{Gal}(k_S/k_S^+)$. At last, from the description of $H^2(G_S)$ by the sequence (2), $H^2(G_S(k^+))$ and $H^2(G_S(k))^+$ are isomorphic. In the case of Demuškin groups, we can make a more precise statement, see [1, Prop. 10]:

Proposition 7 *If $G_S(k)$ is a Demuškin group, then $G_S(k^+)$ is also a Demuškin group, and*

$$H^2(G_S(k)) = H^2(G_S(k))^+.$$

If we restrict to Demuškin groups of local type, the previous proposition has the following converse, see [1, Th. 2]:

Theorem 2 *Let k/k^+ be an extension of CM type, such that $G_S(k^+)$ is a Demuškin group , and suppose that there exists $v \in S(k)$ such that $\mu_p \subset k_v$. Let us also denote by v the place of k^+ dividing v. Then $G_S(k)$ is a Demuškin group if and only if the following conditions hold:*

(i) $\mu_p \subset k_v^+$

(ii) $S(k^+) = \{v\}$ *and* $|S(k)| = 1 + \delta$

(iii) $X'(k)^- = 0$

In this case, $G_S(k)$ and $G_S(k^+)$ are both Demuškin groups of local type.

We now derive some consequences of Theorem 2. We suppose first that $\mu_p \subset k$. Then $X'(k)^- = X'(k)_\omega$. Hence, by using Theorem 1 and Proposition 7, we obtain the following result, see [1, Cor. 1]:

Corollary 1 *(compare with [4, Proposition 5.4]) Let k/k^+ be an extension of CM type, such that k contains μ_p. Then the following conditions are equivalent:*

1. *$G_S(k)$ is a Demuškin group.*
2. *$G_S(k^+)$ is a Demuškin group of local type, and $|S(k^+)| = 1$.*
3. *$S(k^+)$ has only one element v, $\mu_p \subset k^+$, v does not split in k^+_∞/k^+ and $X'(k)^- = 0$.*

In this case we have $G_S = G_S^w = G_w$, where w is one of the two places of k dividing v.

Example. Let $p = 3$, let $k^+ = \mathbb{Q}\sqrt{d}$ be a real quadratic field, and $k = \mathbb{Q}(\sqrt{-3}, \sqrt{d})$. In the previous section, we gave the first values of d such that $G_S(k^+)$ is a Demuškin group of local type. We obtain:

$G_{S_3}(\mathbb{Q}(\sqrt{-3}, \sqrt{d}))$ is a Demuškin group for d = 6, 15, 33, 51, 69, 78, 87, 114, 159, 177...

We suppose now that $\mu_p \not\subset k$. If $G_S(k^+)$ is a Demuškin group, and if the condition (ii) of the Theorem holds, then we have $|S(k_\infty)| = 1$. And under this hypothesis, the conditions $X'(k_\infty)^- = 0$ and $\mathrm{Cl}_S(k)^- = 0$ are equivalent, because $\mathrm{Cl}_S(k)^- = X'(k_\infty)^-_\Gamma$, see [5, Lemma 13.15] for an analogous result. Hence we obtain the following

Corollary 2 *Let k/k^+ be an extension of CM type such that k does not contain μ_p, and suppose that there exists $v \in S(k)$ such that k_v contains μ_p. Then the following conditions are equivalent:*

1. *$G_S(k)$ is a Demuškin group.*
2. *$S(k) = \{v\}$, $\mu_p \subset k_v^+$, $G_S(k^+)$ is a Demuškin group and $Cl_S(k)^- = 0$.*

In this case $G_S(k) = G_S^v(k)$, but $G_S(k) \neq G_v(k)$.

The last assertion of the Corollary comes from the fact that the Euler-Poincaré characteristic $-r_2(k)$ of $G_S(k)$ and the Euler-Poincaré characteristic $-n_v(k)$ of $G_v(k)$ are different.

Example. Let $p = 3$, and let us find the bi-quadratic fields k satisfying Corollary 2. The real maximal subfield $k^+ = \mathbb{Q}\sqrt{d}$, where d is a square free integer, must be of type given by Proposition 6. k must have a unique place v dividing 3, and the inertia group of v in $k/\mathbb{Q}$ is non trivial (because $k^+/\mathbb{Q}$ is ramified) and cyclic, by class field theory. Hence the inertia field k^v is an imaginary (otherwise k would be real) quadratic field: $k^v = \mathbb{Q}\sqrt{-d'}$, where d is a square free integer congruent to 1 (mod 3), because 3 remains prime in k^v. Let $\tilde{k} = \mathbb{Q}\sqrt{-dd'}$ be the other quadratic subfield of k. We have canonically

$$\mathrm{Cl}_S(k)^- \simeq \mathrm{Cl}_S(k^v) \oplus \mathrm{Cl}_S(\tilde{k})$$

Furthermore $\mathrm{Cl}_S(k^v) = \mathrm{Cl}(k^v)$ because the prime 3 is principal in k^v, and $\mathrm{Cl}_S(\tilde{k}) = \mathrm{Cl}(\tilde{k})$ because the prime ideal dividing 3 in $\tilde{k}$ has order 1 or 2 in $\mathrm{Cl}(\tilde{k})$, $\tilde{k}/\mathbb{Q}$ being totally ramified at 3. Hence we obtain the following

Proposition 8 *Let $p = 3$ and $k = \mathbb{Q}(\sqrt{d}, \sqrt{-d'})$, where d and d' are square free integers such that $d \equiv -3 \pmod 9$ and $d' \equiv 1 \pmod 3$. Then G_S is a Demuškin group if and only if the following conditions hold:*

1. *$G_S(\mathbb{Q}\sqrt{d})$ is a Demuškin group.*
2. *$Cl(\mathbb{Q}\sqrt{-d'}) = Cl(\mathbb{Q}\sqrt{-dd'}) = 0$.*

Let m be the square free integer such that $\tilde{k} = \mathbb{Q}\sqrt{-m}$. The following table gives the value of m when $m < 500$. The first line and the first column give respectively the first values d' and d satisfying the congruences of Proposition 8, such that 3 does not divide the class number of $\mathbb{Q}\sqrt{-d'}$ and $G_S(\mathbb{Q}\sqrt{d})$ is a Demuškin group (See the list given in the previous section). $\not m$ means that 3 divides the class number of $\tilde{k}$. The table shows that $G_S(k)$ is a Demuškin group, with $k = \mathbb{Q}(\sqrt{d}, \sqrt{-d'})$, except the cases $(d', d) = (37, 6)$ or $(7, 33)$.

$d\downarrow \overset{d'}{\rightarrow}$	1	7	10	13	19	22	34	37	43	46
6	6	42	15	78	114	33	51	$2\cancel{2}2$	258	69
15	15	105	6	195	285	330				
33	33	$2\cancel{3}1$	330	429		6				
51	51	357					6			
69	69	483								6
78	78		195	6		429				

References

[1] M. Arrigoni, *Representation of Demuškin groups*, Tokyo Metropolitan Univ. Math. Preprint Series, 1995 n° 9.

[2] T. Fukuda, *Iwasawa λ-invariants of imaginary quadratic fields*, J. of the College of Industrial Technology, Nihon Univ. 27 (1994), 35–88, Corrigendum to appear in ibid.

[3] K. Iwasawa, *On $\mathbb{Z}_\ell$-extensions of algebraic number fields*, Annals of Math. 98 (1973), 243–326.

[4] L. V. Kuz'min, *Local extensions associated with ℓ-extensions with given ramification*, Math. USSR Izvestija 9-4 (1975), 693–726.

[5] L. C. Washington, *Introduction to Cyclotomic Fields*, Graduate Texts in Math. 83, Springer, 1982.

[6] K. Wingberg, *On Galois groups of p-closed algebraic number fields with restricted ramification II*, J. reine. angew. Math. 416 (1991),187–194.